Methodology of Wound Volume Estimation from Scanner Data

R. R. Martin
COMSC, UWC, Wales / DEE, UFMA, Brazil

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1 Assumptions

- Separate colour and depth map images of the wound and surrounding area of skin have been obtained, and these are registered.
- The depth map has a coordinate system such that x and y values are more or less exactly known, and the depths measured are in the z direction, where any errors occur.
- The depth values may or may not lie on a regular grid.

2 Method

2.1 Segmentation

- The colour image is used to segment the boundary of the area representing the wound [1, 2, 3, 4].
- Patients of different racial origins will be targeted by the system. The method of segmentation of the wound using skin colour will need to work for a range of original skin colours, and wound colours.
- One method is to find a linear combination of red, blue and green which gives the greatest discrimination between skin and wound colours. Ideally, a whole range of images would be taken from differing patients, and manual segmentation of skin and wounds performed. We would then have 2 sets of pixels, which were known to belong to either skin or wound. A principal components analysis may then give us an axis corresponding to the linear combination best separating the groups of pixels. A threshold value along this axis may be used to delineate the wound.
- An alternative possibility is to try to estimate the “normal” skin colour for each patient, by for example taking an image known to only contain healthy skin, and averaging the pixel values for that image. The absolute value of the difference of this value and the image containing the wound will set pixels on normal skin to low values, leaving the wound pixels with higher values.
- Having basically identified wound pixels, we will need to eliminate background pixels (outside the body completely) if any. Also, we will need to remove pixels corresponding to hairs and small blemishes such as moles, with differing values from areas of normal skin. On a local scale this can perhaps be done by several rounds of median filtering. Other morphological operations can then be used to fill in small gaps remaining in wound and skin blemish areas.

- This should separate the wound and possibly several other skin blemishes. A floodfilling routine can be used to find the largest area which will be assumed to correspond to the wound. (In practice, might the wound be several discrete areas? What are the medical requirements in such a case?)

2.2 Original skin surface estimation

- The area including the wound, and a "reasonable" area surrounding it, are covered by an x-y grid where each grid cell contains many original pixels.

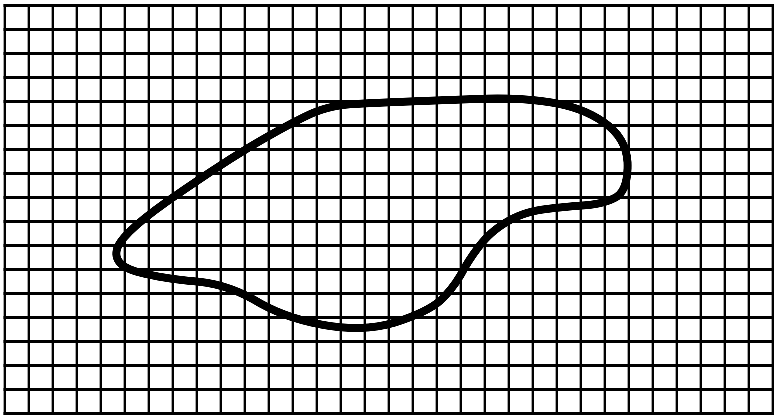

Figure 1: Grid covering good skin and wound

- How is the grid size chosen? There is a trade off between speed of computation, and accuracy of the final result. Do we use the same grid size inside the wound as outside it? Probably a finer grid will be appropriate inside the wound as the bottom surface of the wound may be irregular.
- In each cell, the 3D point with median z value is chosen as a representative point. (Exceptionally, a cell may have no points). We need to treat cells containing the boundary between wound and non-wound carefully. We must select 2 representative points, one outside the wound, for surface fitting, and one inside the wound for volume estimation.

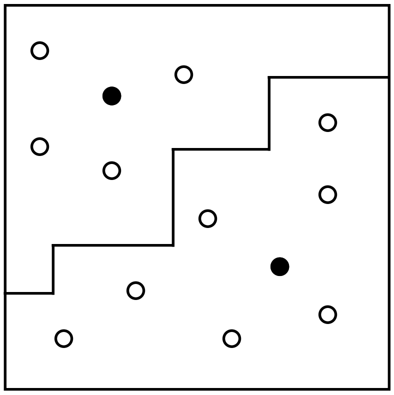

Figure 2: Representative points in a cell inside and outside the wound

- A copy of the cell structure is made which just includes those cells outside the wound.

- "Fitting with ignore areas" [5] is done to the points belonging to cells outside the wound, to give a surface $z = f(x, y)$ which estimates where the skin would have been before the wound was present. (We may need to subdivide the cell structure for a good fit).
- In principle we want a functional which is a compromise between goodness of fit to the depth data, and to a smooth surface, one in which integrated curvature is minimised. In the $z = f(x, y)$ form, this is a nonlinear problem, but it can be approximated by the ignore areas functional $\sum_i (z(x_i, y_i) - z_i)^2 + \gamma \int\int z_{xx} + 2z_{xy} + z_{yy} dxdy$ as shown later.
- How is γ chosen? There is a trade off between producing a good fit to the range data and producing a smooth surface. If γ is used carefully, it will be able to smooth some of the noise in the range data; note also that using median selection will also help to reduce the effects of noise.
- Probably z should be described in terms of control values and a Bernstein or B-Spline basis, but what order and how many control points should be used? We will need to normalise the range of x and y to 0 to 1 if the Bernstein basis is used, or a suitable (uniform?) set of knot values should be chosen if B-Splines are used. Note that the surface fitted should be slightly larger than the extent for which we have x and y values if a Bézier surface is used as we do not have any precise boundary curves.
- Should we adaptively adjust order until the range data fit within a given tolerance? This will be relatively easier if we use a B-Spline formulation.
- The formulae for least squares must be computed according to the appropriate case as shown in the following section. Certain values in the least squares computation are integrals depending only on the basis functions; these can be computed beforehand. The other terms depend on the data values (Note that if x_i and y_i were on a regular grid, other values could be precomputed, but this will not be possible if we use the median smoothing approach outlined above.)

2.3 Volume estimation

- For each cell inside the wound, we take the x and y values of its representative point, and estimate the original skin height s at that point. The wound volume is estimated as the sum over all cells inside the wound of (cell area in x-y plane) $\times$ $(s - z)$. We must multiply the area of the cell by a fraction to allow for how much of it is inside the wound when it is a boundary cell containing pixels both inside and outside the wound.
- If a cell inside the wound is empty (contains no depth value), we will have to estimate an s value for that cell as the average s value of its non-empty neighbouring cells. In an extreme case this may have to be done recursively.

3 Details

3.1 Functional to be minimised

In the original ignore areas version, it is suggested that the functional

$$\sum_i w_i \left(\mathbf{r}(u_i, v_i) - \mathbf{p}_i\right)^2 + \gamma \int\int r_{uu} + 2r_{uv} + r_{vv} dudv$$

is minimised. Here, we assume that the w_i are taken to be 1 for simplicity and that the surface is written in the form

$$\mathbf{r} = \mathbf{r}(u, v, z(u, v)) = (x, y, z(x, y)).$$

Assuming we want to minimise the sum of the squares of the principal curvatures in this integral, we have

$$k_1^2 + k_2^2 = 4M^2 - 2K = 4\left(\frac{En + Gl - 2Fm}{2(EG - F^2)}\right)^2 - 2\frac{ln - m^2}{EG - F^2}$$

where M is the mean curvature and K is the Gaussian curvature, and the coefficients of the first and second fundamental forms are $E = 1 + z_x^2$, $F = z_x z_y$, $G = 1 + z_y^2$; $l = z_{xx}/H$, $m = z_{xy}/H$, $n = z_{yy}/H$ and $H = \sqrt{1 + z_x^2 + z_y^2}$. To make the least-squares problem tractable (linear) we make the approximation that to first order $z_x = z_y = 0$, in which case, after simplification,

$$k_1^2 + k_2^2 = z_{xx}^2 + 2z_{xy}^2 + z_{yy}^2$$

giving the functional to be minimised as stated in the previous section.

3.2 Least squares problem

The functional to be minimised is

$$\sum_i (z(x_i, y_i) - z_i)^2 + \gamma \int\int z_{xx} + 2z_{xy} + z_{yy} dxdy$$

where $z(x, y)$ is assumed to be a tensor product surface in the form

$$z(x, y) = \sum_{j,k} p_{jk} B_j(x) B_k(y);$$

the B_i are basis functions and the p_{jk} are control values. Minimising this functional with respect to the choice of these control values requires

$$\frac{\partial F}{\partial p_{jk}} = 0 \quad \forall j, k.$$

Thus the j,k^{th} equation is

$$0 = -\sum_i z_i B_j(x_i) B_k(y_i) + \sum_i \sum_{m,n} p_{mn} B_m(x_i) B_n(y_i) B_j(x_i) B_k(y_i) +$$
$$\gamma \int\int \sum_{m,n} p_{mn} (B''_m(x) B_n(y) B''_j(x) B_k(y) + 2B'_m(x) B'_n(y) B'_j(x) B'_k(y) + B_m(x) B''_n(y) B_j(x) B''_k(y)) dxdy$$

or

$$\sum_{m,n} p_{mn} (A_{jkmn} + \gamma I_{jkmn}) = \sum_i z_i B_j(x_i) B_k(y_i)$$

where the A_{jkmn} depend on the data but the basis integrals I_{jkmn} just depend on the basis functions and can be evaluated beforehand:

$$A_{jkmn} = \sum_i B_m(x_i) B_n(y_i) B_j(x_i) B_k(y_i)$$

$$I_{jkmn} = \int\int B''_m(x) B_n(y) B''_j(x) B_k(y) + 2B'_m(x) B'_n(y) B'_j(x) B'_k(y) + B_m(x) B''_n(y) B_j(x) B''_k(y) dxdy$$

(The A_{jkmn} could also be evaluated beforehand if the x_i and y_i values were known in advance, for example if the data were on a regular grid).

This gives us a total of jk linear equations in the unknowns p_{mn}.

We need to normalise the ranges of x and y to $[0, 1]$ if we are using Bézier basis functions.

3.3 Basis integrals

This section states without proof how to calculate the basis integrals I_{jkmn} above for two specific cases which may be of use in the wound volume estimation problem, cubic Bézier basis functions, and uniform second-order B-spline basis functions. These results were obtained using *Mathematica.*

3.3.1 Cubic Bézier basis functions

From the definition, I_{jkmn} is separable into one dimensional integrals as follows:

$$\begin{aligned} I_{jkmn} &= \int_0^1 B''_m(x)B''_j(x)dx \int_0^1 B_n(y)B_k(y)dy + \\ & \int_0^1 B_m(x)B_j(x)dx \int_0^1 B''_n(y)B''_k(y)dy + \\ & 2\int_0^1 B'_m(x)B'_j(x)dx \int_0^1 B'_n(y)B'_k(y)dy, \end{aligned}$$

from which we only need values for the integrals

$$i1_{mn} = \int_0^1 B_m(\sigma)B_j(\sigma)d\sigma$$

$$i2_{mn} = \int_0^1 B'_m(\sigma)B'_j(\sigma)d\sigma$$

$$i3_{mn} = \int_0^1 B''_m(\sigma)B''_j(\sigma)d\sigma$$

Expressed in matrix form we have

$$i1 = \begin{pmatrix} 1/7 & 1/14 & 1/35 & 1/140 \\ 1/14 & 3/35 & 9/140 & 1/35 \\ 1/35 & 9/140 & 3/35 & 1/14 \\ 1/140 & 1/35 & 1/14 & 1/7 \end{pmatrix}$$

$$i2 = \begin{pmatrix} 9/5 & -9/10 & -3/5 & -3/10 \\ -9/10 & 6/5 & 3/10 & -3/5 \\ -3/5 & 3/10 & 6/5 & -9/10 \\ -3/10 & -3/5 & -9/10 & 9/5 \end{pmatrix}$$

$$i3 = \begin{pmatrix} 12 & -18 & 0 & 6 \\ -18 & 36 & -18 & 0 \\ 0 & -18 & 36 & -18 \\ 6 & 0 & -18 & 12 \end{pmatrix}$$

We do not give I_{jkmn} in full here for reasons of space but they can be computed trivially from the results above.

3.3.2 Uniform second order B-Spline basis functions

We can split I_{jkmn} again into $i1$, $i2$ and $i3$ as in the Bézier case (with suitably different limits of integration). The corresponding results are expressed slightly differently:

$$i1[1,1] = 11/20, \quad i1[1,2] = 13/60, \quad i1[1,3] = 1/120;$$

and

$$i1[i,j] = i1[1, j-i+1], \quad i1[i,j] = i1[j,i], \quad i1[i,j] = 0 \text{ if } |i-j| > 2.$$

$$i2[1,1] = 1, \quad i2[1,2] = -1/3, \quad i2[1,3] = -1/6;$$

and

$$i2[i,j] = i2[1, j-i+1], \quad i2[i,j] = i2[j,i], \quad i2[i,j] = 0 \text{ if } |i-j| > 2.$$

$$i3[1,1] = 6, \quad i3[1,2] = -4, \quad i3[1,3] = 1;$$

and

$$i3[i,j] = i3[1, j-i+1], \quad i3[i,j] = i3[j,i], \quad i3[i,j] = 0 \text{ if } |i-j| > 2.$$

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